A Lesson on Sequences and Series (Precalc)

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Intro to Sequences

Definition (What is a SEQUENCE?)

In math, a **sequence** is just what you would normally call a **list**.

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You could have a sequence of objects in your house:
Chair, cat, food, door, pumpkin, tv, . . .

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- **1, 3, -5, 5, -4,. . .**

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- **2, 4, 6, 8,. . .**

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- **1, 4, 9, 16,...**

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- **2, 4, 6, 8,. . .** (Positive even numbers!)
- **1, 4, 9, 16,. . .** (Squares!)

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YES! The important thing is that it's not just a jumbled mess of numbers, but a **LIST** of numbers with an **ORDER**.

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The sequences

$$\mathbf{1,\ 3,\ \text{-}5,\ 5,\ \text{-}4,\ldots}$$
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$$\mathbf{1,\ 4,\ 9,\ 16,\ldots}$$

are all **infinite sequences**.

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are all **infinite sequences**. (Why? The ellipsis = ... = dotdotdot)
The sequence of objects in our house, although seemingly VERY long, does not actually go on forever! So it is a **finite sequence**.

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How do we do math with sequences?

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Each element of the list is called a **TERM** of the sequence.

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$$☺_1, ☺_2, ☺_3, ☺_4, \ldots \qquad \text{(If it's infinite)}$$

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$☺_1, ☺_2, ☺_3, ☺_4, \ldots$ (If it's infinite)
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All that matters is the **INDEX**.

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$☺_1, ☺_2, ☺_3$ (If it's finite with three terms)

All that matters is the **INDEX**. Let's give it a shot!

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1, 4, 9, 16,...

3, 1, 1, 1

10, 5, 0

First Sequence:

Intro to Sequences

How do we do math with sequences?

Let's write out a few sequences:

-1, 1, -2, 2, -3, 3,...
1, 4, 9, 16,...
3, 1, 1, 1
10, 5, 0

First Sequence:

$$a_1 = -1$$

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-1, 1, -2, 2, -3, 3,. . .
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First Sequence:

$$a_1 = -1$$
$$a_2 = 1$$

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-1, 1, -2, 2, -3, 3,. . .
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First Sequence:

$$a_1 = -1$$
$$a_2 = 1$$
$$a_3 = -2$$

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-1, 1, -2, 2, -3, 3,...
1, 4, 9, 16,...
3, 1, 1, 1
10, 5, 0

First Sequence:

$$
\begin{aligned}
a_1 &= -1 \\
a_2 &= 1 \\
a_3 &= -2 \\
a_4 &= 2 \\
&\vdots
\end{aligned}
$$

Intro to Sequences

How do we do math with sequences?

Let's write out a few sequences:

$$-1, 1, -2, 2, -3, 3, \ldots$$
$$\mathbf{1, 4, 9, 16, \ldots}$$
$$3, 1, 1, 1$$
$$10, 5, 0$$

Second Sequence:

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Let's write out a few sequences:

-1, 1, -2, 2, -3, 3,. . .
1, 4, 9, 16,. . .
3, 1, 1, 1
10, 5, 0

Second Sequence:

$$a_1 = 1$$

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-1, 1, -2, 2, -3, 3,. . .
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Second Sequence:

$$a_1 = 1$$
$$a_2 = 4$$

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1, 4, 9, 16,. . .
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10, 5, 0

Second Sequence:

$$a_1 = 1$$
$$a_2 = 4$$
$$a_3 = 9$$

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-1, 1, -2, 2, -3, 3,. . .
1, 4, 9, 16,. . .
3, 1, 1, 1
10, 5, 0

Second Sequence:

$$
\begin{aligned}
a_1 &= 1 \\
a_2 &= 4 \\
a_3 &= 9 \\
a_4 &= 16 \\
&\vdots
\end{aligned}
$$

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How do we do math with sequences?

Let's write out a few sequences:

-1, 1, -2, 2, -3, 3,. . .
1, 4, 9, 16,. . .
3, 1, 1, 1
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Third Sequence:

Intro to Sequences

How do we do math with sequences?

Let's write out a few sequences:

-1, 1, -2, 2, -3, 3,. . .
1, 4, 9, 16,. . .
3, 1, 1, 1
10, 5, 0

Third Sequence:

$$a_1 = 3$$

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How do we do math with sequences?

Let's write out a few sequences:

$$
\begin{gathered}
\text{-1, 1, -2, 2, -3, 3,}\ldots \\
1,\ 4,\ 9,\ 16,\ldots \\
\mathbf{3,\ 1,\ 1,\ 1} \\
10,\ 5,\ 0
\end{gathered}
$$

Third Sequence:

$$
\begin{aligned}
a_1 &= 3 \\
a_2 &= 1
\end{aligned}
$$

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Let's write out a few sequences:

-1, 1, -2, 2, -3, 3,. . .
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Third Sequence:

$$a_1 = 3$$
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(Remember you don't need ellipsis for a finite sequence!)

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Fourth Sequence:

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Fourth Sequence:

$$a_1 = 10$$

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When we try to describe a sequence, we want to come up with a **GENERAL FORMULA** for its terms.

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When we try to describe a sequence, we want to come up with a **GENERAL FORMULA** for its terms.
The first type of formula is an **EXPLICIT FORMULA**. This formula uses the **INDEX** explicitly as the variable, which is why we cared so much before about how our sequence is written.

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The first type of formula is an **EXPLICIT FORMULA**. This formula uses the **INDEX** explicitly as the variable, which is why we cared so much before about how our sequence is written.
Sometimes doing this is **VERY HARD**, so don't get discouraged - all you can do is play around with it!

Explicit Formulas for Sequences

Writing Explicit Formulas

$$-1, 1, -2, 2, -3, 3, \ldots$$

First Sequence:

Explicit Formulas for Sequences

Writing Explicit Formulas

-1, 1, -2, 2, -3, 3,. . .

First Sequence:
Notice that the pattern is clearer if we separate even and odd terms

$$\begin{array}{ll} a_1 = -1 & a_2 = 1 \\ a_3 = -2 & a_4 = 2 \\ a_5 = -3 & a_6 = 3 \end{array}$$

Explicit Formulas for Sequences

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So what counts as a general formula?

Explicit Formulas for Sequences

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$$\begin{array}{ll} a_1 = -1 & a_2 = 1 \\ a_3 = -2 & a_4 = 2 \\ a_5 = -3 & a_6 = 3 \end{array}$$

So what counts as a general formula? This is a perfect description:

$$a_n = \begin{cases} -\left(\frac{n+1}{2}\right) & \text{if } n \text{ odd} \\ \frac{n}{2} & \text{if } n \text{ even} \end{cases}$$

Explicit Formulas for Sequences

Writing Explicit Formulas

$$\mathbf{1,\ 4,\ 9,\ 16, \ldots}$$

Second Sequence:

Explicit Formulas for Sequences

Writing Explicit Formulas

$$\mathbf{1,\ 4,\ 9,\ 16, \ldots}$$

Second Sequence:
This is the sequence of squares! Meaning the first term is 1^2, the second term is 2^2, the third term is 3^2, and so on.

Explicit Formulas for Sequences

Writing Explicit Formulas

1, 4, 9, 16,. . .

Second Sequence:
This is the sequence of squares! Meaning the first term is 1^2, the second term is 2^2, the third term is 3^2, and so on.
To describe this mathematically, we can write the general formula

$$a_n = n^2.$$

Explicit Formulas for Sequences

Writing Explicit Formulas

$$3,\ 1,\ 1,\ 1$$

Third Sequence:

Explicit Formulas for Sequences

Writing Explicit Formulas

3, 1, 1, 1

Third Sequence:
This sequence has no real pattern, so we can just write it explicitly as

$$a_1 = 3, a_2 = 1, a_3 = 1, a_4 = 1.$$

Explicit Formulas for Sequences

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Third Sequence:
This sequence has no real pattern, so we can just write it explicitly as

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Or if we want to use a **PIECEWISE FORMULA** like we did for the first sequence, we could write it as

$$a_n = \begin{cases} 3 & \text{if } n = 1 \\ 1 & \text{if } n = 2, 3, 4 \end{cases}$$

Explicit Formulas for Sequences

Writing Explicit Formulas

$$\mathbf{10, 5, 0}$$

Fourth Sequence:

Explicit Formulas for Sequences

Writing Explicit Formulas

$$\mathbf{10,\ 5,\ 0}$$

Fourth Sequence:
This sequence only has three terms but there's still certainly a pattern!

Explicit Formulas for Sequences

Writing Explicit Formulas

10, 5, 0

Fourth Sequence:
This sequence only has three terms but there's still certainly a pattern!

$$a_n = 15 - 5n \quad \text{if } n = 1, 2, 3.$$

Recursive Formulas for Sequences

Writing Recursive Formulas

Instead of writing out a sequence as $a_1, a_2, a_3, \ldots$ every time, we usually just write a_n or $\{a_n\}$ to denote a sequence with terms a_n.

Recursive Formulas for Sequences

Writing Recursive Formulas

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Instead of defining a sequence $\{a_n\}$ **EXPLICITLY** in terms of n, we can also define it **RECURSIVELY** in terms of its other terms.

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To figure out any term, you need to figure out all terms before it AND you need to know the **INITIAL CONDITIONS (ICs)**, where we define the first few terms of the sequence.

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Examples

The sequence defined **RECURSIVELY** by $a_n = 2a_{n-1}$ with **INITIAL CONDITION** $a_1 = 1$ is

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Examples

The sequence defined **RECURSIVELY** by $a_n = 2a_{n-1}$ with **INITIAL CONDITION** $a_1 = 1$ is

$$1, 2, 4, 8,$$

Recursive Formulas for Sequences

Writing Recursive Formulas

Instead of writing out a sequence as $a_1, a_2, a_3, \ldots$ every time, we usually just write a_n or $\{a_n\}$ to denote a sequence with terms a_n.
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Sequence Examples

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SEQUENCE : $8, 10, 12, 14, \ldots$

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Sequence Examples

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Two Special Types of Sequences

Arithmetic and Geometric Sequences

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That's it! That's how they are related.

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ANSWER: A **SERIES** is what we get by adding up all the terms of a **sequence**.

That's it! That's how they are related.

Series Examples

Let's take some previous sequences and turn them into series!

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SEQUENCE: 3, 1, 1, 1

SERIES: 3 + 1 + 1 + 1

Intro to Series

Summation Notation

When we had a sequence $a_1, a_2, a_3, \ldots$ and we didn't want to keep writing it all out every time, we created the shorthand $\{a_n\}$.

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$$\sum_{n=1}^{k} a_n.$$

The sum symbol is a capital greek Sigma, so we also sometimes call it **SIGMA NOTATION**.

Intro to Series

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$$\sum_{n=1}^{k} a_n.$$

The sum symbol is a capital greek Sigma, so we also sometimes call it **SIGMA NOTATION**.
We call n the **INDEX**, the value $n = 1$ the **LOWER LIMIT** of the sum, and the value $n = k$ the **UPPER LIMIT** of the sum. Note the upper limit can be infinity, $k = \infty$, when we sum infinite sequences! The term a_n inside the sum is called the **SUMMAND**.

Intro to Series

Series Examples

Let's take some previous sequences and turn them into series in **SUMMATION NOTATION**!

Intro to Series

Series Examples

Let's take some previous sequences and turn them into series in **SUMMATION NOTATION**! The way we do this is by replacing a_n with its **EXPLICIT FORMULA**.

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If the explicit formula is complicated enough, we just leave it as a_n with the reference to an explicit formula.

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Intro to Series

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SEQUENCE: 3, 1, 1, 1

SERIES: $\sum_{n=1}^{4} a_n$

Intro to Series

Wait a second - the first two sums just keep getting bigger, right?

If a series adds up to a single number, then we say it **CONVERGES**. Otherwise, we say it **DIVERGES**.

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A series **DIVERGING** could look like a few different things - the sum could get bigger and bigger and go off to ∞ or $-\infty$, or it could just bounce around and never settle to a single number.

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A series **DIVERGING** could look like a few different things - the sum could get bigger and bigger and go off to ∞ or $-\infty$, or it could just bounce around and never settle to a single number.
Let's grab EVERY sequence we've listed in this lesson and see whether its series converges or diverges!

Intro to Series

The Great Series Check!

Sequence: 1, 3, -5, 5, -4,. . .

Series: Diverges, because the sequence was randomly chosen.

Intro to Series

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Sequence: 1, 3, -5, 5, -4,. . .

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Sequence: 2, 4, 6, 8,. . .

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Sequence: 1, 4, 9, 16,. . .

Series: Diverges to infinity.

Sequence: -1, 1, -2, 2, -3, 3,. . .

Series: Diverges, because it never lands on one number

Adding up its terms one-by-one gives -1, 0, -2, 0, -3, 0, . . . , which never settles on a single number.

Intro to Series

The Great Series Check!

Sequence: 3, 1, 1, 1

Series: Converges to $3 + 1 + 1 + 1 = 7$.

Intro to Series

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Series: Converges to $10 + 5 + 0 = 15$

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Sequence: 1, 2, 4, 8, 16, 32, ...

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Sequence: 1, 2, 4, 8, 16, 32, ...

Series: Diverges to infinity.

Sequence: 1, 1, 2, 3, 5, 8, ...

Series: Diverges to infinity.

Sequence: $7, -\frac{7}{5}, \frac{7}{25}, -\frac{7}{125}, \ldots$

Series: Converges to $\frac{35}{6}$

Series Formulas

What patterns are we seeing?

Whoa, what happened with that last series?

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The sequence $7, -\frac{7}{5}, \frac{7}{25}, -\frac{7}{125}, \ldots$ gives us the series

$$\sum_{n=1}^{\infty} 7\left(-\frac{1}{5}\right)^n$$

which indeed converges to $\frac{35}{6}$.

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which indeed converges to $\frac{35}{6}$.

The reason THIS particular infinite series converges can be explained by the BIG FINALE of this whole topic!

Series Formulas

BIG FINALE

The reason why we single out the arithmetic and geometric sequences specifically is that their series have particularly clean formulas.

Arithmetic Series

If the sequence $\{a_n\}$ is arithmetic, then we call the series $\sum_{n=1}^{k} a_n$ an arithmetic series.

Series Formulas

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Because arithmetic sequences have such a nice form $a_n = a_1 + (n-1)d$, the series nicely evaluates as:

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$$\sum_{n=1}^{k} a_n = k\left(\frac{a_1 + a_k}{2}\right) = \frac{k}{2}(2a_1 + d(k-1)).$$

Series Formulas

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$$\sum_{n=1}^{k} a_n = \frac{a_1(1-r^k)}{1-r}.$$

Series Formulas

Infinite Series

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Infinite Series

If we try to take an infinite sum of an **ARITHMETIC SEQUENCE**, then it will **DIVERGE** unless it's $a_n = 0$.

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only if $|r| < 1$. If this is the case, then multiplying by r every time shrinks the terms quick enough where the sum converges!

Series Formulas

Series Summary

If a_n is an arithmetic series with common difference d, then

$$\sum_{n=1}^{k} a_n = k\left(\frac{a_1 + a_k}{2}\right) = \frac{k}{2}(2a_1 + d(k-1)).$$

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If a_n is a geometric sequence with common ratio r, then

$$\sum_{n=1}^{k} a_n = \frac{a_1(1-r^k)}{1-r}.$$

If we have infinitely many terms (so $k \to \infty$), then $r^k \to 0$, so

$$\sum_{n=1}^{k} a_n = \frac{a_1}{1-r}.$$

Series Formulas

Final Example

Looking at the sequence $7, -\frac{7}{5}, \frac{7}{25}, -\frac{7}{125}, \ldots,$ we see that $a_1 = 7$ and $r = -\frac{1}{5}$, so using our formula, the series converges to

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$$\frac{a_1}{1-r} = \frac{7}{1+\frac{1}{5}} = \frac{7}{\frac{6}{5}} = \frac{35}{6}.$$

Thank You!!

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Please message me with any questions or comments on this lesson and I will get back to you as soon as possible!